A spatio-temporal Stochastic Pattern Generator for use in ensemble prediction and ensemble data assimilation

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Introduction

Model errors (i.e. imperfections in the forecast-model equations and boundary conditions) are a substantial contributor to forecast errors and therefore need to be adequately represented in ensemble prediction and ensemble data assimilation systems. In this study, we propose a Stochastic Pattern Generator (SPG) for model-error simulation. The SPG produces 2D and 3D spatio-temporal pseudo-random fields with tunable spatial and temporal length scales and meaningful space-time interactions. The SPG operates on a limited-area domain. The technique is implemented as a Fortran program freely available from https://github.com/gayfulin/SPG.

Methodology

The SPG is based on a stochastic differential equation in time with a pseudo-differential spatial operator:

$$\left(\frac{\partial}{\partial t} + \frac{U}{\lambda}\sqrt{1-\lambda^2\Delta}\right)^3 \xi(t,\mathbf{s}) = \alpha(t,\mathbf{s}). \tag{1}$$

Here ξ is the random field in question (the output of the SPG), t is time, $\mathbf{s}$ is the spatial vector, Δ is the spatial Laplacian, $\alpha(t,\mathbf{s})$ is the white in time and space noise, λ is the scalar that controls the spatial length scale, and U is the velocity-dimensioned parameter that controls the temporal length scale.

Equation (1) is numerically solved using a finite difference scheme in time and a spectral scheme in space. For motivation, derivations, properties, numerical scheme, simulation results, and other details, see Tsyrulnikov and Gayfulin (2016b) and Tsyrulnikov and Gayfulin (2016a).

Results

Figure 1 displays the spatio-temporal correlation function for an SPG random field. One can see that the larger is the time lag, the broader are the spatial correlations. This is a manifestation of the non-separability of the spatio-temporal correlations often observed in the real world (e.g. Cressie and Huang, 1999, Fig.8). More specifically, the spatio-temporal structure of the SPG fields is scale dependent, so that longer spatial scales ‘live longer’ than shorter spatial scales, which “die out” quicker. This desirable property was called “proportionality of scales” in (Tsyroulnikov, 2001). The SPG is embedded into the Fortran code of the limited-area meteorological non-hydrostatic model COSMO (Baldauf et al., 2011). Results of a numerical experiment are displayed in Fig.2.

Conclusions

A generator of spatio-temporal pseudo-random Gaussian fields that satisfy the “proportionality of scales” property is presented. The generator is a third-order in time stochastic differential equation with a pseudo-differential spatial operator defined on a limited area 2D or 3D domain in the Cartesian coordinate system. The spatio-temporal covariances are non-separable. The spatial covariance functions of the generated fields belong to the Matérn class. A spectral-space numerical solver is implemented. The generator is tested with the COSMO model as a source of additive spatio-temporal perturbations to the forecast model fields. The SPG can be used to generate spatio-temporal perturbations of the model fields (in the additive or multiplicative or other mode), as well as the boundary conditions.

Bibliography

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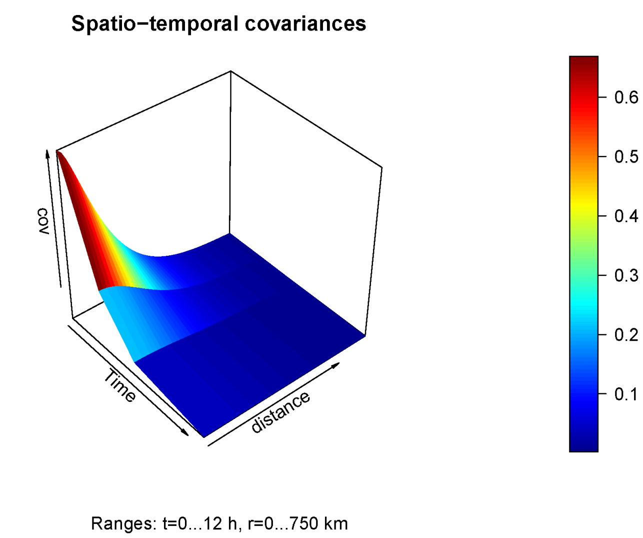

Figure 1: The theoretical spatio-temporal SPG correlation function.

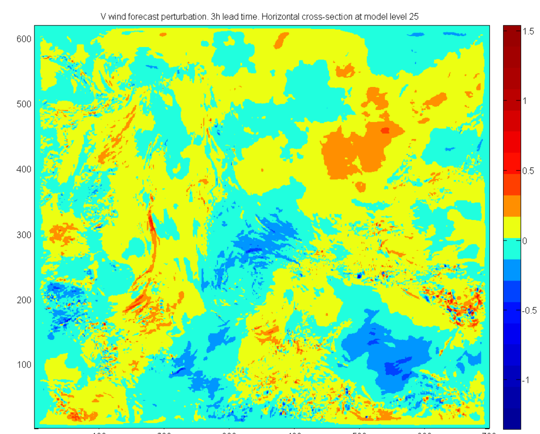

Figure 2: COSMO forecast V-wind perturbation field in response to the additive SPG perturbations of temperature, pressure, and both wind components after 3 hours of time integration (bottom). The SPG perturbations were added every 15 minutes.

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