Moist-entropic vertical adiabatic lapse rates: the standard cases and some lead towards inhomogeneous conditions.

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In a recent paper, Marquet (2011) proposed a new moist-entropic potential temperature θ_s, linked to the second law of thermodynamics through its full equivalence to the specific moist entropy *s*, i.e. with consideration of the 'dry air' and 'water species' subparts of the atmospheric parcel, of specific content q_d plus the total water specific content $q_t=1-q_d=q_v+q_l+q_i$. The likely advantage of θ_s with respect to earlier proposals is that it is both Lagrangian-conservative and tractable in mixing processes. Given the obvious links between any kind of potential temperature and vertical adiabatic lapse rates, we elected to do the analytical computation of such lapse rates on the basis of parcels keeping θ_s constant. This can be done without approximation only for the cases (i) of no condensed phase at all (named here 'non-saturated', rather than using the ambiguous 'dry') and (ii) of fully saturated conditions (named here 'saturated'). Beware that we shall consider here only saturation with respect to liquid water, the extension to ice water conditions being rather straightforward. In fact the results presented below were obtained with the even more ambitious goal to look at the vertical stability under any (neutral or not) conditions, see Marquet and Geleyn (2012). But we shall concentrate here on the adiabatic lapse rates, going for this to further details than in the above-mentioned paper.

Despite the apparent complexity of the analytical formulation for θ_s

$$\theta_s = T\,(p_0/p)^{R_d/c_{pd}}\; e^{-\left(\frac{L_{vap}(T)\,q_l}{c_{pd}\,T}\right)}\; e^{(\Lambda_r q_t)} \left(\frac{T}{T_r}\right)^{\lambda q_t} \left(\frac{p}{p_r}\right)^{-\kappa\delta q_t} \left(\frac{r_r}{r_v}\right)^{\gamma q_t} \times \frac{(1+\eta\, r_v)^{\kappa(1+\delta q_t)}}{(1+\eta\, r_r)^{\kappa\delta q_t}} \,,$$

the results in terms of adiabatic lapse rates are beautifully compact. Indeed, the formulations (11) and (16) to (18) in Marquet and Geleyn (2012) can be written as:

- in the non-saturated case $\Gamma_{ns} = g/c_p\,;$

- in the saturated case $$\Gamma_{sw} = (g/c_p)\,\frac{1+\left(\dfrac{L_{vap}(T)\,r_{sw}}{R_d T}\right)}{1+\left(\dfrac{R}{c_p}\right)\left(\dfrac{L_{vap}(T)}{R_v T}\right)\left(\dfrac{L_{vap}(T)\,r_{sw}}{R_d T}\right)}\,.$$

The first formulation, with c_p obviously depending on the parcel's composition (q_d, q_v, q_l, q_i), was expected. But the second one differs from the 'classical' ones advocated by Durran and Klemp (1982) or Emmanuel (1994), which both contain an additional term in the lower case and do not return to g/c_p when eliminating the aspects linked to condensation. Probably because of the very general character of θ_s, our saturated result on the contrary allows identifying as sole specific multipliers, the non-saturated adiabatic lapse rate, the 'full parcel kappa' R/c_p and the Clausius-Clapeyron factor. The results thus sound logical and especially consistent, since they take into account in a fully logical way the dependence of L_{vap}, c_p and R with the temperature and composition of the moist air.

Now, if trying to get away from our extreme cases (non-saturated and saturated), one notices that, owing to their simplicity, the transition between both formulations is equivalent to just replacing the constant "*1*" by (R/c_p) $[\, L_{vap} / (R_v\ T)\,]$. But the second value may also be reorganised in the shape $[\, L_{vap} / (c_p\ T)\,] / (R_v/R)$. The latter expression is nothing else (Marquet and Geleyn, 2012, Appendix F) than the ratio of the impacts of water vertical transport on buoyancy, between saturated conditions (when only latent heat release acts) and non-saturated conditions (when only density-linked expansion acts).

Hence, defining by C a weighting factor (which may, in a certain sense, be considered as the proportion of an air parcel being in saturated conditions), it is natural to express a <u>*generalised shape for the vertical adiabatic lapse rate*</u>, now under non-homogeneous conditions:

$$F(C) = 1 + C\left[\frac{L_{vap}(T)}{c_p\,T}\,\frac{R}{R_v} - 1\right], \qquad D_C = \frac{L_{vap}(T)\,r_{sw}}{R_d\,T}, \qquad r_{sw} = \frac{\varepsilon\, e_{sw}(T)}{p - e_{sw}(T)} ,$$

$$\Rightarrow \quad \boxed{\Gamma(C) = (g/c_p)\,\frac{1 + D_C}{1 + F(C)\,D_C} \quad .}$$

Two remarks are needed here:

- Alike in the above-mentioned earlier publications, our definition of the saturation point corresponds to reversible conditions (i.e. at constant q_t) and not to the irreversible ones of 'permanent exact saturation'. In the second case it is q_{sw} which would depend only on pressure and temperature, in the first case this happens for r_{sw}.
- If D_C had been written with r_v replacing r_{sw}, $\Gamma(C)$ would still have been compatible with its two extreme boundary conditions. But it is precisely in order to get the more logical situation of a term independent of the air parcel's composition multiplying $F(C)$ that we chose the above D_C formulation for $\Gamma(C)$, expressed in terms of r_{sw}.

Concerning the second remark, one may even make D_C more compact with the help of the Clausius-Clapeyron relationship:

$$D_C = \frac{L_{vap}(T)}{R_v\,T}\left[\frac{e_{sw}(T)}{p - e_{sw}(T)}\right] = \frac{T}{p - e_{sw}}\,\frac{de_{sw}}{dT} \quad !$$

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<u>References</u>:

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