The reduced grid with variable latitude resolution for the global semi–Lagrangian numerical weather prediction model

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It is well known that the parameterizations of sub–grid processes in the forecast model may work incorrectly on the non–isotropic grid. To avoid such an obstacle we developed an approach to construct the longitude–latitude reduced grid with variable latitude resolution suitable for semi–Lagrangian finite difference models.

The quasi–uniform grid on the Earth surface imposes some constraints on the steps within the region under consideration. Therefore the iterative method of grid generation implies following restrictions on the grid step ratio: $\max(\Delta\phi/(\Delta\lambda\cos(\phi)), \Delta\lambda\cos(\phi)/\Delta\phi) \leq C(\phi) + \delta C$ and $\Delta_j\phi/\Delta_{j+1}\phi \leq R$. Beginning with the uniform (in both latitude and longitude) grid we construct a one–dimensional latitude mesh which is used for the reduced grid generation. Than we diminish δC and continue this procedure.

Our technique of latitude mesh generation is based on the physical analogy between a simplex mesh and the truss structure [2] where meshpoints are nodes of the truss. Assuming an appropriate force–displacement function for the bars in the truss we determine the equilibrium of this system. The latitudinal grid steps with high resolution in the vicinity of Novosibirsk (in latitude 55° North) are shown in Fig. 1a.

The main goal of the reduced grid construction method is to minimize the total number of the grid nodes at the fixed upper limit ϵ_Ψ for the sum of the r. m. s. interpolation error of a given function f_k^0 $(k = 1, \cdots, n_k)$:

$$\Psi = \sum_{j=1}^{n_\phi} \sum_{k=1}^{n_k} \int_0^{\pi/2} |f_k(\lambda, \phi_j) - f_k^0(\lambda, \phi_j)|^2 \cos(\phi_j) \mathrm{d}\lambda, \qquad \Psi \leq \epsilon_\Psi \tag{1}$$

Here $f_k^0 = \cos\left(\phi^* \frac{n_\phi}{10}\right) \cos\left(\lambda^* \frac{n_\lambda}{10} + \frac{\pi}{2}\frac{k}{n_k}\right)$, where (ϕ^*, λ^*) are coordinates in the rotated system with North pole at $(55°, 45°)$ with respect to regular coordinate system (ϕ, λ).

Equation (1) is solved numerically for each value of ϵ_Ψ and the grid obtained in such a way we call as the optimal reduced grid. It should be noted that the properties of such a grid substantially depend on the function f_k^0. Normalized values of the grid steps $\Delta\phi$ and $\Delta\lambda$ as a function of latitude ϕ are shown in Fig. 1b. Small disturbances on the lower curve (the longitudinal step) are due to the restriction on the number of longitudinal grid points $n_\lambda(\phi)$ because it is the product of $2^n \cdot 3^m \cdot 5^l$ (where $n \geq 2, m \geq 0, l \geq 0$).

We carried out a number of shallow–water tests [4] that involve both the solid–body rotation of a cosine bell around the sphere through the poles and two cases of the deformational flow tests. In all cases our method of grid generation was found to be promising. It should be noted that the error of the numerical solution obtained on the reduced grid with uniform latitude resolution is somewhat higher in comparison with that presented in [1].

This method will be used for construction of the grid for the new version of the weather prediction SL–AV global model [3] with conservative semi–Lagrangian scheme. Latitudinal derivatives in this model are calculated in the space of longitudinal Fourier coefficients, so that the reduced grid can be implemented.

Advantage of our method is that it allows us to take into account various details of the weather prediction model and to apply additional restrictions on the grid. This work was supported by the RFBR grant 10-05-01066.

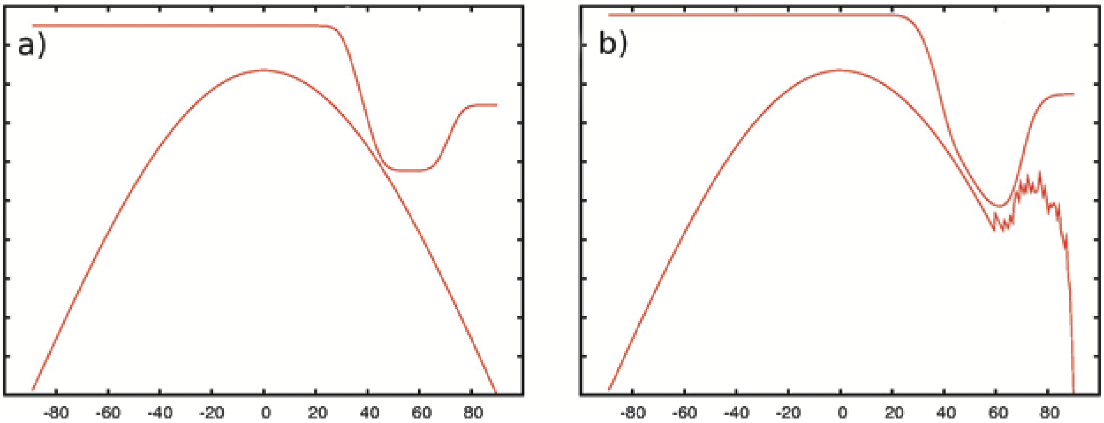

Figure 1: Normalized latitudinal (upper curve) and longitudinal (lower curve) steps after first iteration (a) and when the iteration convergence criterion is achieved (b).

References

[1] Fadeev R. Yu. Reduced latitude–longitude grid constructing for the global numerical weather prediction. – Russ. Meteor. and Hydrol., 2006, N 4, pp. 5–20.

[2] Persson P.–O., Strang G., A Simple Mesh Generator in MATLAB. SIAM Review, Volume 46 (2), pp. 329-345, June 2004

[3] Tolstykh M. A. Semi–Lagrangian high resolution model of atmosphere for numerical weather prediction. – Russ. Meteor. and Hydrol., 2001, N 4, pp. 1–9.

[4] Williamson D. L., Drake J.B., et al. A standart test set for numerical approximations to the shallow water equations in spherical geometry. – J. Comput. Phys., 1992, vol. 102, pp. 211–224.