A FORMAL APPROACH FOR SMOOTHING ON VARIABLE-RESOLUTION GRID
Part I: Design and application for Cartesian coordinates

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Global climate models with variable resolution are used to improve the representation of regional scales over an area of interest, avoiding the nesting issues of the limited-area models and reducing the computational costs compared to global uniform high-resolution models. To address some potential problems associated with the stretching and anisotropy of the computational grid, a general convolution filter operator was developed. The main feature of this filter is to locally remove scales shorter than a user-prescribed spatially varying length scale.

Over non-uniform grids, the only scales that can be represented over the entire domain are those with length scales larger than or equal to twice the maximum grid spacing $\Delta x_{\max}$. In a stretched-grid model, it would be possible to remove anisotropic features associated with fine-scale structure of the mesh in only one dimension, such as in the arms-of-the-cross region surrounding the high-resolution area of interest. This may be an effective means of controlling aliasing of fine-scale features while they exit the high-resolution region and enter in the low-resolution part of the domain. In this case, the numerical filtering operator could suitably remove the unwanted small scales outside the uniform high-resolution area.

The convolution operator was chosen to design the filtering formula. For a signal ψ, the filtered value $\overline{\psi}$ is: $\overline{\psi}(x)=(\psi * w)(x)=\int_{-\infty}^{\infty}\psi(x)\cdot w(x-s)ds$. Using the convolution theorem and taking the Fourier transform of the initial ψ and the filtered $\overline{\psi}$ fields to evaluate the ratio of their spectral amplitudes, one finds that the required weighting function w is the inverse Fourier transform of the desired response function (e.g. Surcel 2005). The numerical filtering operator that will remove all the waves that are not correctly represented outside the high-resolution region has a spectral response that corresponds to keeping unchanged the large scales with wavenumber smaller than $a=2\pi/2\Delta x_{\max}$ (corresponding to wavelengths larger than $2\Delta x_{\max}$ that are resolved everywhere on the grid), and removing entirely small scales with wavenumber larger than a chosen value of $b\le 2\pi/2\Delta x_{\min}$, with a gradual transition in between to reduce Gibbs' phenomenon. The convolution theorem then gives $w(d)=\frac{\pi}{2}\cdot\frac{\sin ad+\sin bd}{d}\cdot\frac{1}{\pi^2-d^2(b-a)^2}$, where $d=x-s$. Although the formal definition of the convolution exists for continuous space, its practical application needs a definition using a discrete set of points. Also, the presence of non-vanishing values over the entire domain is problematic because of excessive computational costs. Given that the values become small after some distance from the origin, truncating the weighting function after some distance can result in an important reduction in computational costs. While the truncation distance is an important parameter for the precision of the filter, the distance between the wavenumbers a and b affects also the choice of the truncation distance and subsequently the precision of the filter. A weighting function corresponding to an abrupt change in the spectral response contains large oscillations, and needs a large truncation distance, while a more gradually varying response function gives rise to a narrow weighting function, and thus to a much smaller acceptable truncation distance, in order to approximate adequately the theoretical response.

The application of the convolution filter for a variable resolution domain is tested considering a periodic 1D stretched grid $[0,2\pi]$ with a stretching factor $S=\Delta x_{\max}/\Delta x_{\min}\cong 4$. This grid contains 60 to 64% from the total number of grid points in the high-resolution area, even if the area represents only 1/3 from the entire domain. The filter is tested using the 1D test-function as $\psi(x)=\psi_l(x)+\psi_n(x)$, where ψ_l is a large-scale wave representing the physical signal that is properly represented on the entire domain, and ψ_n is a small-scale wave representing the noise which is chosen to be zero in the low-resolution part of the domain, gradually increased in the stretching areas, and is maximum in the high-resolution area of the domain. The skill of the filter is quantitatively evaluated by comparing the filtered solution $\overline{\psi}$ with the expected analytical solution ψ_l, using two scores: the normalized root-mean square error (*NRMS*) that is computed between the filtered solution and the expected analytical solution, and the normalized conservation ratio (*NCR)* calculated as the mean error between the filtered and unfiltered solution (Surcel and Laprise, 2010).

A first example presents a test function containing a noise with wavenumber $k_n = 2\pi/4\Delta x_{\min}$. The convolution filter used a weighting function w_1 defined by $a = 2\pi/2\Delta x_{\max}$ and $b = 1.5a$; in this case a truncation distance $d_{\max} = 5\,\Delta x_{\max}$ was adequate to completely remove the noise while maintaining the large-scale signal. The effects of using different weighting functions was studied employing the same test function and two different weighting functions: w_1 ($a = 2\pi/2\Delta x_{\max}$; $b = 1.5a$) and w_2 ($a = 2\pi/2\Delta x_{\max}$; $b = 2a$). The *NRMS* score (Fig. 1b) shows that the noise is completely removed if an adequate truncation distance is used and this distance is shorter for the filter with a more gradual response (w_2). In this case we note also a better conservation expressed by a *NCR* score, which approaches zero for a truncation distance larger than $10\Delta x_{\min}$ as it can be seen in Fig. 1c.

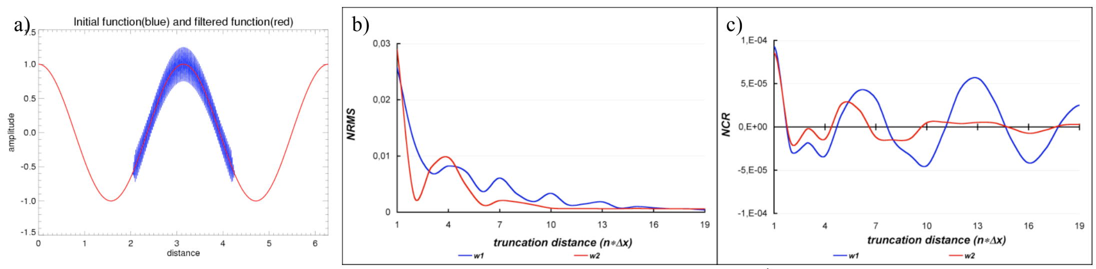

Figure 1. (a) The initial test function (blue) containing a noise with wavenumber $k_n = 2\pi/4\Delta x_{\min}$ and the filtered field (red) are represented on a grid with $S \cong 4$. The convolution uses the weighting function w_1 and a truncation distance $d_{\max} = 5\,\Delta x_{\max}$. The *NRMS* (b) and the *NCR* (c) scores as a function of the truncation distance, for two different weighting functions.

The formal approach developed in 1D is generalized for two-dimensional domains. The 2D convolution uses a weighting function that is the product of two 1D functions, similar with those used in one-dimensional case. In practice, the 2D filtered function is obtained conveniently by successive applications of 1D filter in each direction. It is important to reiterate that in our proposed convolution filter approach, the weights are calculated using physical distances rather than grid-point indices.

Similarly to our tests in 1D, test functions using 2D sinusoidal waveforms were chosen to represent the large-scale signal that will be retained by the filter and the noise that will be removed. The example presented in Fig. 2 shows the initial signal (a) and the filtered function when the convolution uses the weighting function w_1 and truncation distances of $d_{\max} = 10\,\Delta x_{\min}$ (b) and $d_{\max} = 21\,\Delta x_{\min}$ (c).

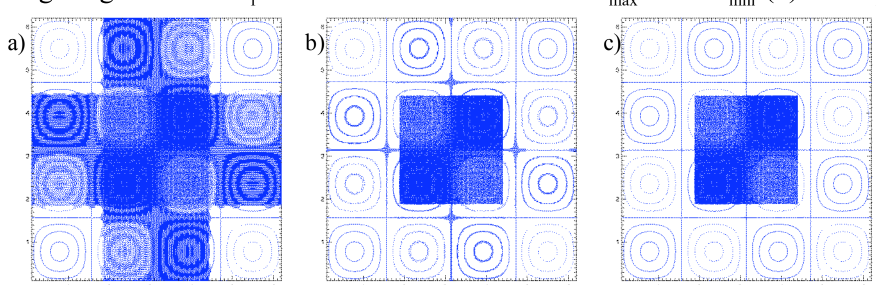

Figure 2: The initial test field on the 2D stretched-grid with $S \cong 4$ (a). The filtered fields after application of a convolution filter with 1D weighting function w_1 and truncation distances of $10\,\Delta x_{\min}$ (b) and $21\,\Delta x_{\min}$ (c).

The proposed approach appears to be a valuable alternative to a conventional grid-point based smoothing operator for stretched-grid models. The filter can be used to render quasi-isotropic fields on variable-resolution grids. A key element of this approach is that the weighting function is based on the physical distance rather than grid point indices. The convolution filter developed in 1D and generalised for 2D Cartesian geometry will next be adapted for polar geometry.

References

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