A three-dimensional conservative semi-Lagrangian scheme (SLICE-3D) for transport problems*

M. Zerroukat, N. Wood and A. Staniforth
Met Office, FitzRoy Road, Exeter EX1 3PB, U.K.

6th February 2006

Last year we reported on the development of an inherently conservative semi-Lagrangian (SLICE) scheme for two-dimensional passive transport in spherical geometry [1]. Since then, the reconstruction part of the scheme has been further developed by introducing the parabolic spline method (PSM), an optimal reconstruction having the minimum norm (or curvature) and best approximation properties [2]. Furthermore an improved grid and sub-grid scale monotonicity filter [2] has also been incorporated into the scheme. This filter is more selective and less damping in the smooth part of the solution than other filters.

A three-dimensional version of SLICE has now been developed in spherical geometry and tested on a variety of flows (see the example of figure 1). SLICE-3D does one sweep of a one-dimensional conservative remapping for each column in the vertical direction. This results in the transfer of mass contained between the original regular concentric spherical shells ($k = 1, ..., K$) to intermediate deformed concentric spherical shells ($k' = 1, ..., K$). This is followed by $(K - 1)$ applications of SLICE-2D around the $K - 1$ concentric deformed spherical annulii. Each application of SLICE-2D also makes multiple sweeps of the same one-dimensional remapping along physical distances around the deformed spherical annulii. It is also worth noting that no knowledge of the complex 3D geometrical details of the individual Lagrangian volumes is required or computed at any stage of the computation. This confers a significant efficiency advantage on the SLICE methodology over fully geometrical remapping algorithms. The resulting scheme makes the possibility of higher-dimensional remapping with high-order reconstructions, without a prohibitive computational cost, a reality.

References

[1] M. Zerroukat, N. Wood, A. Staniforth, SLICE-S: A Semi-Lagrangian Inherently Conserving and Efficient scheme for transport problems on the Sphere, Q. J. R. Meteorol. Soc. 130 (2004) 2649–2664.

[2] M. Zerroukat, N. Wood, A. Staniforth, The Parabolic Spline Method (PSM) for conservative transport problems, Int. J. Numer. Meth. Fluid (in press).

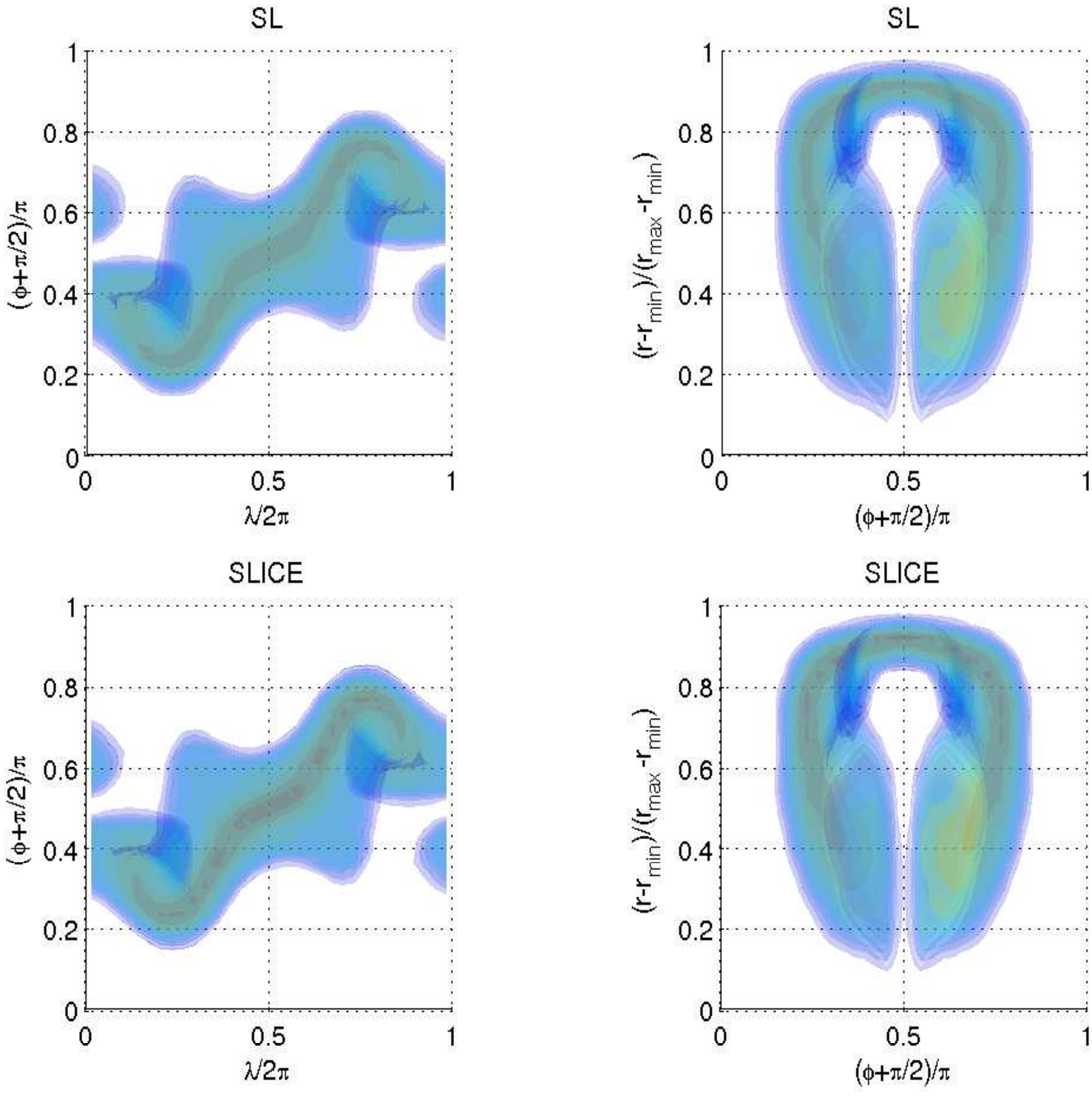

Figure 1: Comparison of results using the SLICE (bottom pair of plots) and standard tri-cubic interpolating Semi-Lagrangian (SL, top pair of plots) algorithms, in a spherical annulus lying between $r_{min} = 1$ and $r_{max} = 2$. A spherical distribution of mass, spanning approximately 1/3rd of the domain in each direction, is initially centred at the equator on the $\lambda = \pi$ meridian. It is then subjected to a flow field given by the superposition of stationary idealisations of a Rossby wave and a Hadley cell circulation. Two views of the solutions at $t = 1$, performed with timestep $\Delta t = 0.1$ and $32 \times 32x32$ control volumes(λ, ϕ, r), are displayed: the left and right pairs are (λ, ϕ) and (ϕ, r) views respectively.