Analysis of the Generalized Vertical Coordinate for the Euler Equations of Atmospheric Motion

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It has been known that for the successful modeling of mesoscale atmospheric events with an essentially vertical structure of motion it is natural to use the full non-hydrostatic system of Euler equations rather than their hydrostatic approximation. It has also been known that the hydrostatic approximation can be constructed with the hydrostatic pressure as a vertical coordinate. Kasahara (1974) tried to summarize the transformation of the vertical coordinate for the governing equations of atmospheric motion. However, he did not discuss the transformation of the vertical momentum equation. Laprise (1992) proposed a form of the full system of non-hydrostatic equations with pseudo-hydrostatic pressure as a vertical coordinate. He affirmed that such approach is consistent and without loss of generality. This approach has been chosen by Bubnova et al. (1995) to build a new non-hydrostatic dynamical kernel of the ALADIN NWP limited-area model. However, the non-hydrostatic ALADIN remained in research mode until the beginning of the 21st century. The research on the non-hydrostatics benefited from the parallel work on the semi-Lagrangian technique of advection which brought into evidence some important issues such as the construction and the use of the vertical acceleration in a hydrostatic model (e.g.: Gospodinov et al. 2002). Benard et al. (2004) advanced significantly in the development of the non-hydrostatic ALADIN based on the Laprise's system.

In this work we attempt to go further than Kasahara (1974) and write the full non-hydrostatic system of Euler equations with generalized vertical coordinate. We attempt to derive the system of Laprise (1992) with pseudo-hydrostatic pressure as a vertical coordinate within the considerations of our theory. We analyze the properties of the system of Laprise (1992) by developing a split system. We attempt to situate the vertical acceleration in the full transformed system and in the system of Laprise (1992). The results of this study suggest that the system of Laprise (1992) is restricted with regards to the full non-hydrostatic system. It is possible to derive a measure of the error, introduced into the system with the particular choice of a vertical coordinate, namely the pseudo-hydrostatic-pressure vertical coordinate. The measure of error for the evolution tendency for the vertical derivative

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of the advecting vertical velocity is given by the following expression:

$$\left(\left(\frac{\partial \mathbf{V}}{\partial x}\right)_{\pi} \cdot \nabla_{\pi}\right) \mathbf{V} + \left(\left(\frac{\partial \mathbf{V}}{\partial y}\right)_{\pi} \cdot \nabla_{\pi}\right) \mathbf{V} + \frac{\partial \mathbf{V}}{\partial \pi} \cdot \nabla_{\pi} \dot{\pi} \ ,$$

where the notation is as in Laprise (1992). By analyzing the split model, it is also possible to be shown that the physical vertical velocity, within the system of Laprise (1992), is a diagnostic value and not a fully independent variable. If we want to build a non-hydrostatic model with a specific vertical coordinate, different from the geometric height, it is essential to define the law of evolution for the advecting vertical velocity which should not be a diagnostic relation of the type of the pseudo-hydrostatic-pressure coordinate of Laprise (1992).

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